Variable Projection Networks for Arrhythmia Detection

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Outline

- Motivation
- Methodology
- Results
- Conclusion

Outline

General motivations

Motivation

Deep learning (DL) approaches

- Data-driven black box models with usually unexplainable output.
- Huge performance gain in many applications.
- Resource (data, computing, memory) intensive methods.

Model-based approaches

- White box models with usually transparent decisions.
- Incorporates domain knowledge via mathematical modeling.
- Tradeoff between performance and interpretability.
- Manageable resource requirements.

Idea: train VP + NNs in a model-driven learning framework

- VP: automatic feature extractor with interpretable parameters
- NN: learn and model non-linear and complex relationships

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Orthogonal transformations

Methodology

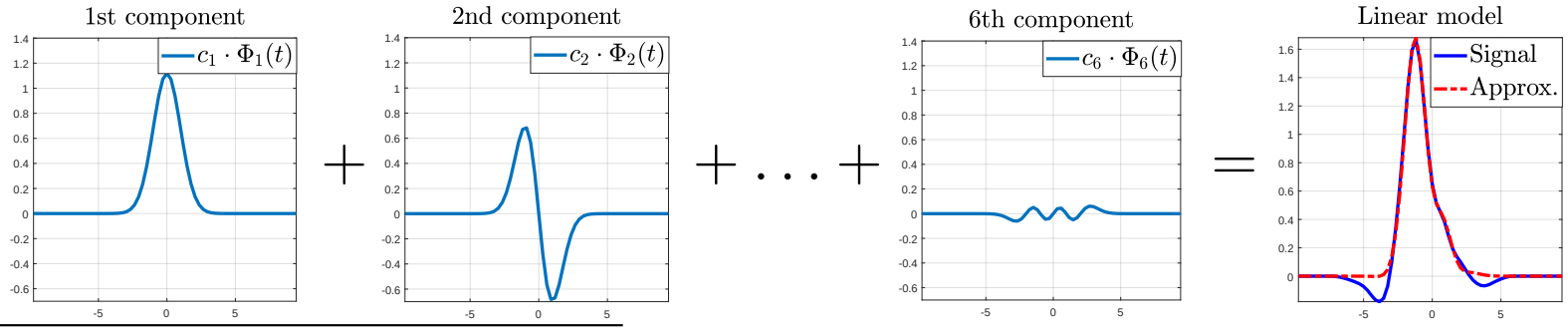
- Linear signal modeling:

$$x \approx \tilde{x} = \sum_{k=0}^{n-1} c_k \Phi_k = \Phi c \qquad (x \in \mathbb{R}^N)$$

- Best approximation problem in Hilbert spaces:
  $\mathcal{S} := \mathrm{span}\{\Phi_0, \Phi_1, \ldots, \Phi_{n-1}\} \subset \mathbb{R}^N$ generated subspace

$$\mathrm{dist}(x, \mathcal{S}) := \min_{y \in \mathcal{S}} \|x - y\|_2 = \|x - \tilde{x}\|_2$$

- Solution to the discrete case (linear least squares):
  - Coefficients vector: $c = \Phi^+ x$
  - Orthogonal projection: $\tilde{x} = P_{\mathcal{S}} x = \Phi \Phi^+ x$
- Orthogonal transformations with system Φ, *e.g.: trigonometric, Hermite [1], etc.*



[1] Sörnmo, L., Börjesson, P.L., Nygards, M. E., Pahlm, O., A method for evaluation of QRS shape features using a mathematical model for the ECG, *IEEE Trans. on Biomed. Eng.*, 1981;28:713–717.

Parameterized orthogonal transformations

Methodology

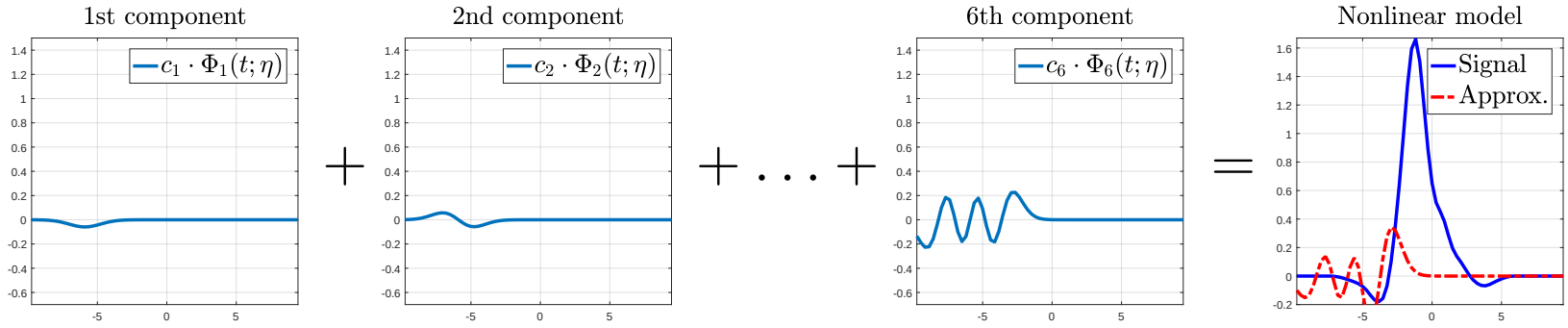
- Nonlinear signal modeling:

$$x \approx \tilde{x} = \sum_{k=0}^{n-1} c_k \Phi_k(\theta) = \Phi(\theta) c \qquad (x \in \mathbb{R}^N)$$

- Variable projection problem [2]:
  $\mathcal{S}(\theta) := \operatorname{span}\{\Phi_0(\theta), \Phi_1(\theta), \ldots, \Phi_{n-1}(\theta)\} \subset \mathbb{R}^N$ generated subspace

$$\min_\theta \operatorname{dist}(x, \mathcal{S}(\theta)) := \min_\theta \|x - \Phi(\theta)\Phi^+(\theta)x\|_2$$

- Solution to the discrete case (nonlinear least squares):
  - Coefficients for a given $\theta \in \mathbb{R}^m$: $c = \Phi^+(\theta)x$
  - Parameterized orthogonal projection: $\tilde{x} = P_{\mathcal{S}(\theta)}x = \Phi(\theta)\Phi^+(\theta)x$
- θ usually represents interpretable quantities, e.g., attenuation, shape parameters.



[2] G. H. Golub, V. Pereyra. The differentiation of pseudo-inverses and nonlinear least squares problems whose variables separate. *SIAM J. on Numerical Analysis*, 1973.

Advantages

- Mathematical model of biological neurons.
- Theoretically approved by the universal approximation theorems.
- Easy to train due to its differentiability.

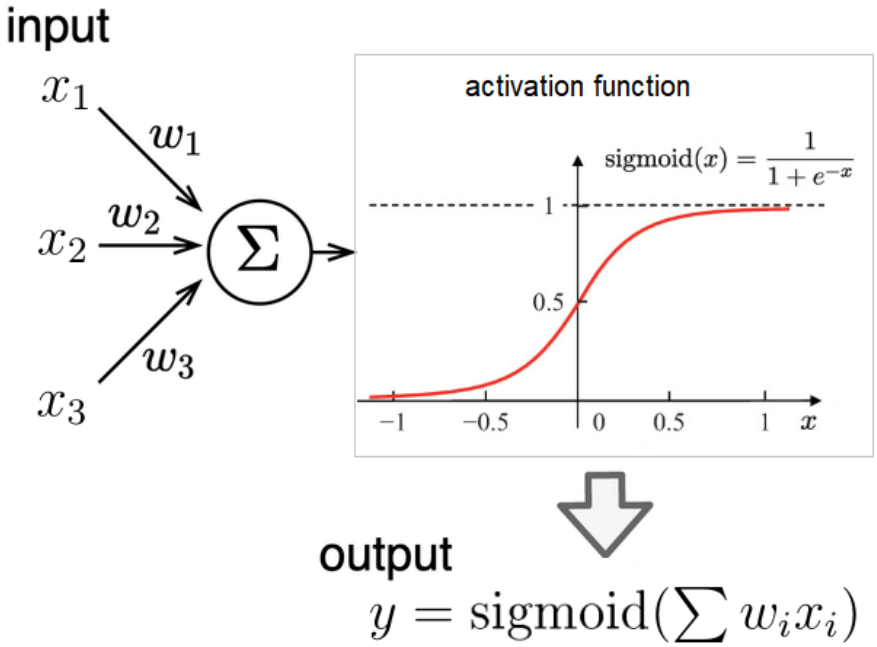


Figure: Artificial neuron model.

Advantages

- Biologically plausible neuron model.
- Power efficiency due to the sparsity of the output spike trains.
- Reduced number of floating point operations due to binary output.

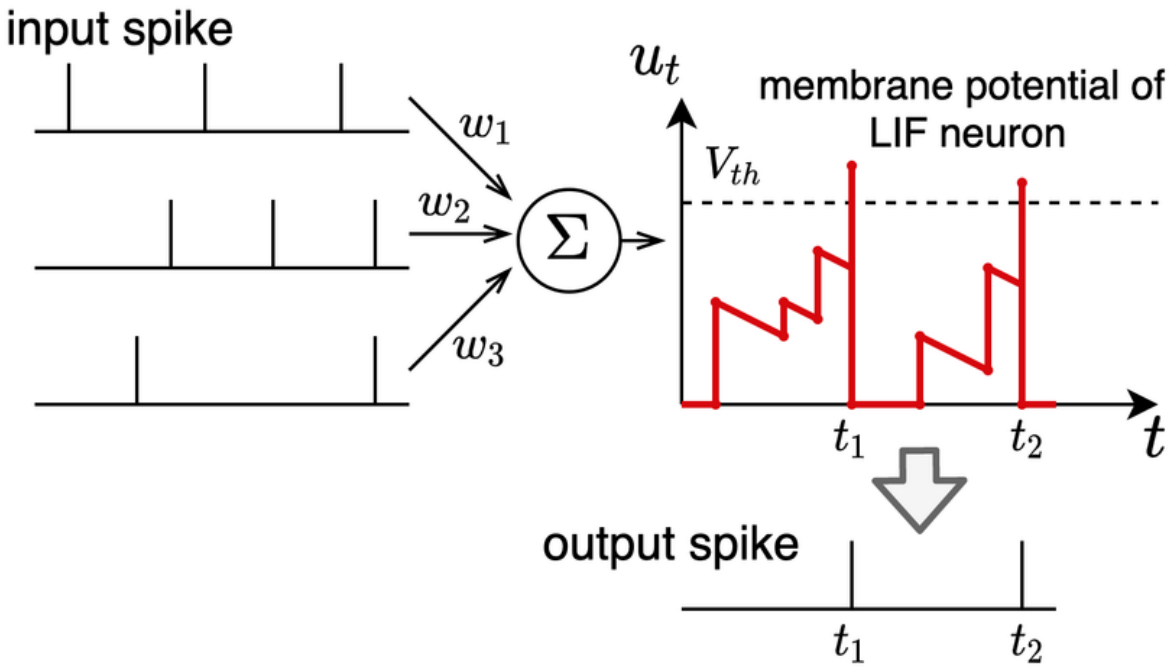


Figure: Leaky integrate-and-fire (LIF) neuron model.

VPNet architecture for spikelike waveforms

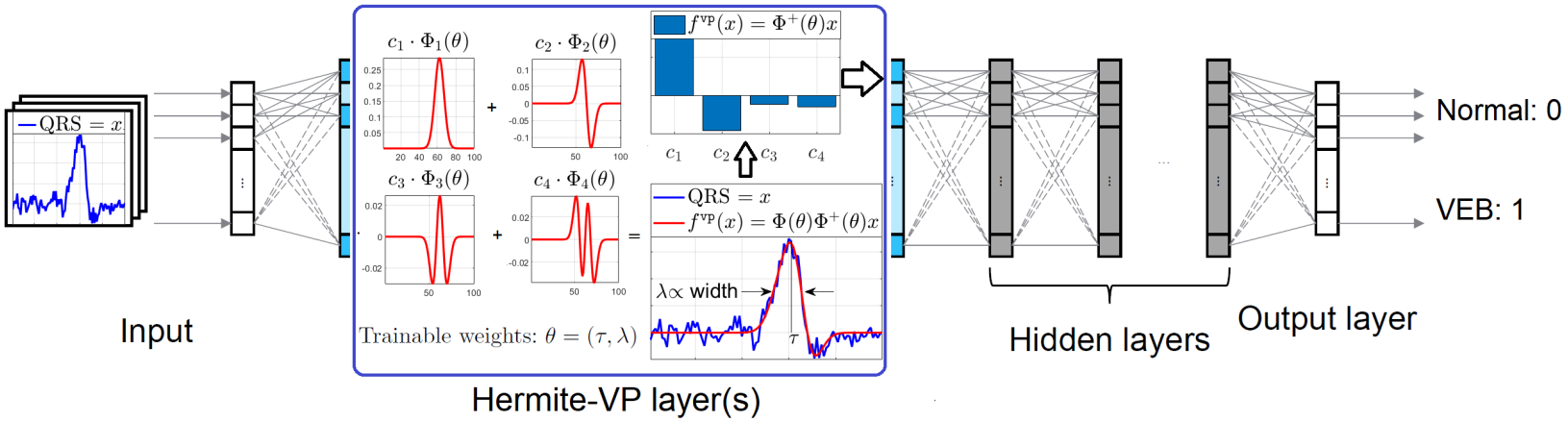
Methodology

- System of Hermite polynomials: $\{h_k \mid k \in \mathbb{N}\}$.
- System of Hermite functions:

$$\Phi_k(t) = h_k(t)/||h_k||_2 \cdot e^{-t^2/2} \qquad (k \in \mathbb{N}).$$

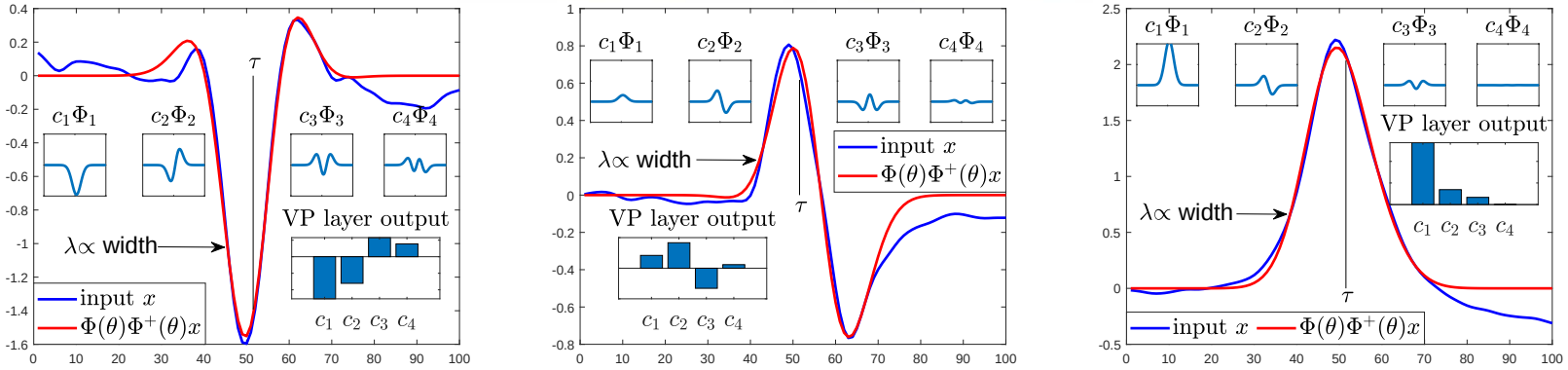
- Nonlinear parameters: dilation and translation $\theta = [\tau, \lambda]^T$
- Parametrization:

$$\Phi_k(\tau, \lambda; t) := \sqrt{\lambda} \cdot \Phi_k\left(\lambda(t - \tau)\right) \quad (t, \tau \in \mathbb{R}, \lambda > 0)$$



VPNet architecture for spikelike waveforms

Methodology



(a) Normal beat (b) Abnormal beat (c) Abnormal beat

Figure: Output of a trained VP layer: for a normal beat (a) and two abnormal beats (b)-(c).

VPNet architectures

- VP + ANN [3]
- VP + SNN [4]

[3] Kovács, P., Bognár, G., Huber, C., Huemer, M., VPNet: Variable Projection Networks, *International Journal of Neural Systems (IJNS)*, pp. 2150054-1–19, 2021.

[4] Kovács, P., Samiee, K., Arrhythmia detection using spiking variable projection neural networks, *Computing in Cardiology (CinC)*, pp. 1–4, 2022.

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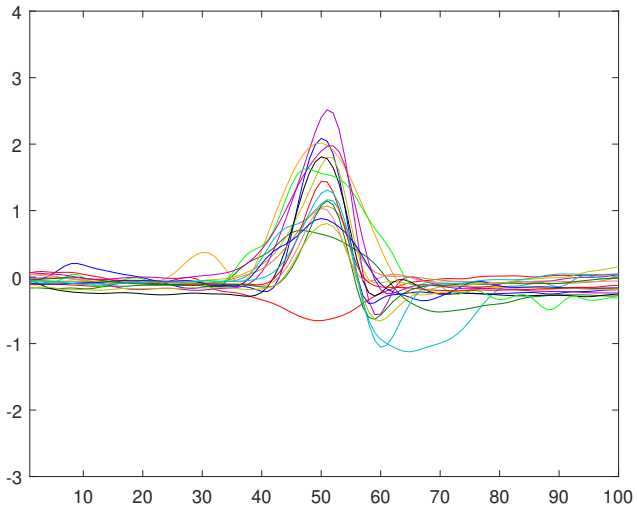
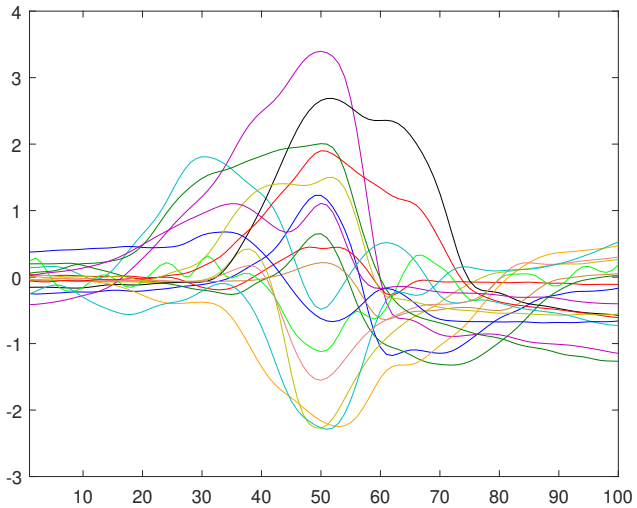
Classification of real ECG heartbeat signals

Results



(a) Normal beats



(b) Ventricular ectopic beats

- PhysioNet MIT-BIH Arrhythmia Database
- Normal $\leftrightarrow$ ventricular ectopic heartbeats
- Balanced dataset: 4260-4260 beats (train), 3220-3220 (test)

Performance evaluation on real ECG data

Results

Table: Overall Classification performances of different NN architectures.

Architecture	$Acc\%$	$Spec\%$	$Sens\%$
ANN	94.32%	95.53%	93.11%
CNN	95.92%	96.09%	95.75%
SNN	95.59%	93.73%	**97.45%**
SCNN	95.42%	95.31%	95.53%
VPANN	96.65%	96.83%	96.61%
VPSNN	96.61%	99.10%	94.13%
VPTSNN	**97.16%**	**99.60%**	94.72%
MVPSNN	94.55%	96.52%	92.58%
VPCSNN	95.61%	98.07%	93.14%

Performance evaluation on real ECG data

Results

Table: Total number of parameters and inference memory size of different NN architectures.

Architecture	# of parameters	memory size
ANN	59010	234 KB
CNN	212610	967 KB
SNN	58880	231 KB
SCNN	376704	1.5 MB
VPANN	39	**0.7 KB**
VPSNN	242	5.3 KB
VPTSNN	242	5.3 KB
MVPSNN	72	2.2 KB
VPCSNN	**26**	1.1 KB

Outline
Conclusion

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Conclusion

- *Summary*
  - Novel model-based NN architecture for 1D signal processing
  - Incorporated domain knowledge, e.g., Hermite-basis for QRS
  - Compact network topology
  - Fast and efficient implementation, e.g., neuromorphic devices
  - Interpretable parameters, e.g., QRS location and width
  - Preliminary results: performance comparable with state of the art with low computational cost.
- *Further research*
  - More experiments: larger datasets, more classes, etc.
  - New fields of applications: visually evoked potentials